Cooperative Radar and Communications Signaling

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This work was sponsored in part by DARPA under the SSPARC program. The views expressed are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.
Topics

• Why is this guy talking about radars?
• What do radars care about?
• What’s the problem?
• How well can you do?
• Where do we go from here?
Topics

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• What do radars care about?

• What’s the problem?

• How well can you do?

• Where do we go from here?
Radars

• That’s that big dish thingy, right?

• Or, the thing the policed used to give me a ticket?

Although Laser Ranging is More Common Now
Simple Radar

- Bounce RF signal off scatterers

- Detect if something is there
  - Lots of hypotheses

- Measure how long it takes for a pulse to return
  - Ranging (and potentially angle and Doppler estimation)
• Radio Detection And Ranging (RaDAR)
  – Let’s be glad that it has transitioned from acronym to word (radar)

• “First” “Radar”
  – Telemobiloscope: name that Christian Hülsmeyer used in his 1904 patent
  – Practically, he could not really do ranging but it would do detection

• Alexander Popov observed multipath effects caused by ships in communications in 1897
  – Invented “sensorless sensing”?
Commercialization of Radars

• Radars are starting to show up everywhere
  – You’ll be wearing radars soon

• Dramatic reduction in costs, size, weigh, and power over the last decade
  – Entering the age of radar on a chip
Vehicular Radars

- Avoid collisions
  - Driver error
  - Self driving cars

- “See” better

- Fuse with other modalities
  - Visible
  - IR
  - Lidar
  - Ultrasonics
Personal Radars

- You will be using, even wearing radars soon
  - Gesture interface

Google ATAP’s Soli
2015 Google I/O
techcrunch.com
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Pulse-Doppler Processing

- Transmit sequence of short “pulses”

- Build matched filter for range and Doppler hypotheses
  - Assume that Doppler is not resolvable by a single pulse

\[ h(\tau, f_D) \propto \int dt \, z(t) \, [s(\tau - t) \, e^{i2\pi f_D t}]^* \]
Radar Waveforms


dB – Chirp

• Standard radar waveform
• Complex tone that ramps

\[ x(t) \propto e^{\frac{i\pi B t^2}{T}} \]

• Constant modulus
• Approximately uniform spectrum

Chirp Waveform

Chirp Spectral Cartoon

Power Spectral Density (linear)
Empowering Digital Modulations

• Can use communications signals for radar
  – Variety of potential concerns
    • Sidelobes
    • Clutter mitigation

Cross Ambiguity Function

• Consider ambiguity function
  – Like a range-Doppler point spread function

\[ \chi(\tau, \delta f) = \int_{-\infty}^{\infty} dt \, s(t) \, s^*(t - \tau) \, e^{i \frac{2\pi}{\tau} \delta f t} \]
Radar Performance Characterization

• Detect targets
  – Function of SINR
  – Presented in terms of receiver operating characteristic (ROC) performance

• Estimate target parameters
  – Limited by the Cramer-Rao Bound
  – Variance $\sim (B^2 \text{SINR})^{-1}$

ROC Performance

![ROC Performance Graph](image)

- $n_s = 1$
- $n_s = 10$
- SNR = 0 dB

Probability of False Alarm

Figure 16.2
Single antenna energy detection probability of false alarm (black) and probability of detection (gray) for a Gaussian signal in the presence of Gaussian noise, assuming an SNR of 0 dB for 10 observations.

Figure 16.3
Single antenna energy detection probability of detection as a function of probability of false alarm for a Gaussian signal in the presence of Gaussian noise, assuming an integrated SNR of 0 dB and 10 dB for 1 (black) and 10 (gray) observations.
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Potential Solutions
From Traditional Radar Perspective

• Don’t give it to them
  – Fighting powerful economic forces
  – U.S. does not control world’s spectrum

Atmospheric Attenuation

- Push radars to X-band and above
  – Many radars are already there
  – Transmit power (complicated trade)
  – Some loss in long range propagation
  – Getting chased by comms again

• Explore radar and radio coexistence
  – DARPA SSPARC efforts

World Telecom
1.6 Trillion/Year
Gartner, Inc. 2014.

Probably
Can’t See
Radar Slice

Radars Versus Radios
Some Similarities

• Emits RF energy

• Receives RF energy

• Translates RF energy into information

• More bandwidth is usually better

• Higher center frequency…
  – Smaller antennas
  – Worse propagation

Actually, This Is an Opportunity
Passive Radar
Not Quite Cooperative Comms and Radar

- Employ existing RF energy to locate scatterers
  - Classic example is TV broadcast signal

- Estimate broadcast signal from direct path

- Estimate scattering environment

![Diagram of Passive Radar](image)

Return From Helicopter

S. Carson, et al., 2006
Examples of Future RF Reuse

• Improve automobile safety
  – Reuse same signals
  – Inter-vehicle communications
  – Collision avoidance

• Reuse future 28-90 GHz and 5G cellular band
  – High data rate communications
  – Cell phone environmental awareness
  – Next generation interfaces
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Heterogeneous System Not Interference!  
*Change the Rules*

- Assume capable nodes (radar/communications)
  - Radios can estimate channels
  - Radars can decode and transmit communications signals
  - Radar waveform is the communications signal

- Estimate performance bounds
  - Mix of information and estimation theory
Crazy Example of Potential Gains

• Use advantaged radar propagation to improve communications
  – Radar as a relay

• Assume propagation
  – Terrestrial comms $\sim$range$^{-4}$
  – Radar-to-comms link $\sim$range$^{-2}$

• Evaluate ratio of capacity

![Graph showing performance improvement with radar range](image)
Our Current Research

• Find fundamental limits on joint radar target estimation and communications performance

• Focus on joint receiver performance as key issue

Critical Assumptions

• Radar return and communications
  – Same frequency allocation
  – Simultaneous

• Radar can decode and mitigate communications signal

• Represent radar performance as rate
  – Target parameters structured random process
  – Note: radar estimation not detection
Communications Information Bound

**Shannon Limit**

- Consider bound on communications on data rate

- Communications rate (b/s) for Gaussian signal and noise
  - Shannon limit

\[ R_{com} \leq B \log_2 \left( 1 + \frac{a^2 P_{com}}{\sigma^2_{\text{noise}}} \right) \]

\[ = B \log_2 (1 + \text{SNR}) \]
Multiple Access Communications Receiver

- Assume radar return and communications
  - Same operating band
  - Simultaneous
  - Assume all single antenna transmitters and receivers

- What is the best joint data information rate?

- This is not our problem
- This is an analogy
Multiple Access Communications Bound

Illustrative Analogy

- Satisfy all bound
  \[ R_1 \leq \log_2(1 + a_1^2 P_1) \]
  \[ R_2 \leq \log_2(1 + a_2^2 P_2) \]
  \[ R_1 + R_2 \leq \log_2(1 + a_1^2 P_1 + a_2^2 P_2) \]

- Fix power of channel for both transmitters

- Find points of bound intersection

  \[ R_2 = \log_2(1 + a_2^2 P_2) \]
  \[ R_1 + R_2 - R_2 = \log_2(1 + a_1^2 P_1 + a_2^2 P_2) - \log_2(1 + a_2^2 P_2) \]
  \[ R_1 = \log_2 \left( \frac{1 + a_1^2 P_1 + a_2^2 P_2}{1 + a_2^2 P_2} \right) \]

  \[ \{ R_1, R_2 \} = \left\{ \log_2 \left(1 + \frac{a_1^2 P_1}{1 + a_2^2 P_2}\right), \log_2(1 + a_2^2 P_2) \right\} \]

  \[ \{ R_1, R_2 \} = \left\{ \log_2(1 + a_1^2 P_1), \log_2 \left(1 + \frac{a_2^2 P_2}{1 + a_1^2 P_1} \right) \right\} \]
Multiple Access Radar/Comms Receiver
Are The Bounds Equivalent?

• Equivalent? No
  – Estimation is not drawn from countable distribution
  – Bound is not achievable

• But, let’s see how close we can get
  – Focus on achievable (inner) bounds

Hint: We Will Apply MUD to Mixed Radar and Communications
Our Approach

• Developed new formalism for analyzing joint radar and communications performance
  – Use communications multiple access channel as motivation

• Employ mix of information and estimation theory

• Construct multiple inner (achievable) bounds

Constructed Novel Estimation Rate Metric

Define Receiver Model

Develop Bounds on Joint Performance

Apply New Tools To System Analysis & Design
• Develop concept of estimation information rate

• Estimate target range (delay)

• Assume delay determined by partially known random process
  – Assume unknown delay is Gaussian

Target Delay

\[ \tau_{m}(k) = \tau_{m,\text{pre}}(k) + n_{\tau,\text{proc}}(k) \]

Target Delay Variance

\[ \sigma^2_{\text{proc}} = \langle n_{\tau,\text{proc}}^2 \rangle \]
Range (Delay) Estimation Uncertainty

- Assume good estimator and reasonable integrated SNR
- Use Cramer-Rao bound to get delay estimation performance bound

\[
\sigma_{\tau; \text{est}}^2 = \left( \frac{1}{8\pi^2 B_{\text{rms}}^2 \text{ISNR}} \right)
\]

\[
B_{\text{rms}}^2 = \frac{\int df f^2 \|X(f)\|^2}{\int df \|X(f)\|^2}
\]

\[
\langle f \rangle = 0
\]
• Invent target estimation rate
  – Assume process variation and estimation error are Gaussian

• Determine estimation information rate by evaluating total and estimation entropies
  – \( R_{est} \sim H_{uncertainty} - H_{est} \)
  – Average number of bits required to encode estimate per unit time

\[
R_{est} \leq \sum_{m} \frac{\delta}{2T} \log_2 \left( 1 + \frac{\sigma_{\tau_m,\text{proc}}^2}{\sigma_{\tau_m,\text{est}}^2} \right)
\]

• Ratio of variances looks like an “SNR”

Like SNR \( \frac{\sigma_{\tau,\text{proc}}^2}{\sigma_{\tau,\text{est}}^2} \)
Our Approach

• Developed new formalism for analyzing joint radar and communications performance
  – Use communications multiple access channel as motivation

• Employ mix of information and estimation theory

• Construct novel multiple inner (achievable) bounds

- Constructed Novel Estimation Rate Metric
- Define Receiver Model
- Develop Bounds on Joint Performance
- Apply New Tools To System Analysis & Design
Three Inner Joint Bounds

Basic Multi-Access Receiver Block

Joint Performance Bounds

• Isolated Sub-Bands
  – What is done traditionally

• Successive Interference Cancelation

• Water-Filling

Note: We Will Present Results for Single Target
Isolated Sub-band Inner Bound

What We Do Now

• Split spectrum $B$ into sub-bands
  – Radar only
  – Communications only

• Bandwidth split determined by $\alpha$

$$B = B_{\text{rad}} + B_{\text{com}}$$

$$B_{\text{com}} = \alpha B$$

**Subband Usage**

- Comms Only
- Radar Only

**Power Density**

**Frequency**

**Isolated Sub-band Bound**

$$R_{\text{com}} \leq \frac{b^2 P_{\text{com}}}{k_B T_{\text{temp}} B_{\text{com}}}$$

**Estimation Rate (b/s)**

$$R_{\text{est}} \leq \frac{B_{\text{rad}}}{2} \log_2 \left( 1 + \frac{2\sigma_{\text{proc}}^2 \gamma^2 B_{\text{rad}}^2 T \|a\|^2 P_{\text{rad}}}{k_B T_{\text{temp}}} \right)^{\delta/(TB_{\text{rad}})}$$
Three Inner Joint Bounds

Joint Performance Bounds

• Isolated Sub-Bands

• Successive Interference Cancelation
  – Operate communications at rate for given radar residual
  – Then, remove communications residuals
  – Operate radar without communications residuals

• Water-Filling
Bound Approach Overview

Successive Interference Cancellation

- Construct novel joint radar/communications approach

- Basic successive interference cancellation (SIC) bound
  - Define radar random process
  - Evaluate estimation error of radar
  - Evaluate estimation information rate
  - Evaluate communications capacity
  - Evaluate SIC point
  - Interpolate between SIC point and communications-only point
Received Signal After Predicted Radar Return Removed

- Received signal combines radar return and communications signal

\[ z(t) = \sqrt{P_{com}} b s_{com}(t) + \sqrt{P_{radar}} \sum_{m=1}^{N} a_m s_{radar}(t - \tau_m) + n(t) \]

- Remove predicted radar return

\[ \tilde{z}(t) = \sqrt{P_{com}} b s_{com}(t) + n(t) + \sqrt{P_{radar}} \sum_{m=1}^{N} a_m [s_{radar}(t-\tau_m) - s_{radar}(t-\tau_m,\text{pre})] \]

- Approximate difference with derivative

\[ \tilde{z}(t) \approx \sqrt{P_{com}} b s_{com}(t) + n(t) + \sqrt{P_{radar}} \sum_{m=1}^{N} a_m \frac{\partial s_{radar}(t - \tau_m)}{\partial t} n_{\tau,\text{proc}} \]

- Characterize “noise” to communications decoder at “radar”

\[ n_{\text{int+n}} = \sqrt{P_{radar}} \left( \sum_{m=1}^{N} a_m \frac{\partial s_{radar}(t - \tau_m)}{\partial t} n_{\tau,\text{proc}} \right) + n(t) \]

By Parseval's Theorem

\[ \sigma^2_{\text{int+n}} = \langle \| n_{\text{int+n}} \|^2 \rangle = P_{rad} \left( \sum_{m=1}^{N} \| a_m \|^2 (2\pi)^2 B_{\text{rms}}^2 \sigma^2_{\text{proc}} \right) + \sigma^2_{\text{noise}} \]
Evaluate SIC Point

- Find maximum communications rate such that the receiver can decode and subtract it in presence of radar return residual

\[ R_{\text{com}} \leq B \log_2 \left[ 1 + \frac{b^2 P_{\text{com}}}{\sigma_{\text{int+n}}^2} \right] = B \log_2 \left[ 1 + \frac{b^2 P_{\text{com}}}{\|a\|^2 P_{\text{rad}} \gamma^2 B^2 \sigma_{\text{proc}}^2 + k_B T_{\text{temp}} B} \right] \]

- Then have ideal radar range estimation

\[ R_{\text{est}} \leq \frac{1}{2} B \log_2 \left[ 1 + \frac{2\sigma_{\tau,\text{proc}}^2 \gamma^2 B (TB) \|a_m\|^2 P_{\text{rad}}}{k_B T_{\text{temp}}} \right]^{\delta/(TB)} \]
Three Inner Joint Bounds

Joint Performance Bounds

• Isolated Sub-Bands

• Successive Interference Cancelation

• Water-Filling
  – Split bands (mixed use and communications only)
  – Split communications power and rate across subbands
  – Operate at SIC point in mixed use subband
Bound Approach Overview

Water-Filling Bound

- Split into two sub-bands
  - Communications only
  - Mixed use
- Optimize comms power use by employing water-filling
Distribute Power By “Water-Filling”

- Optimize comms power/rate between bands
  - Operate mixed-use band at SIC point

\[ B = B_{\text{com}} + B_{\text{mix}} \]
\[ B_{\text{com}} = \alpha B \]
\[ B_{\text{mix}} = (1 - \alpha) B \]

\[ P_{\text{com}} = P_{\text{com,com}} + P_{\text{com,mix}} \]
\[ P_{\text{com,com}} = \beta P_{\text{com}} \]
\[ P_{\text{com,mix}} = (1 - \beta) P_{\text{com}} \]

\[ \mu_{\text{mix}} = \frac{b^2}{\sigma_{\text{int+n}}^2} \]
\[ \mu_{\text{com}} = \frac{b^2}{k_B T_{\text{temp}} B_{\text{com}}} \]

- Define two channels

\[ \beta = \frac{P_{\text{com,com}}}{P_{\text{com}}} = \alpha + \frac{1}{P_{\text{com}}} \left( \frac{\alpha - 1}{\mu_{\text{com}}} + \frac{\alpha}{\mu_{\text{mix}}} \right) \]

when \( P_{\text{com}} \geq \frac{\alpha}{(1 - \alpha) \mu_{\text{mix}}} - \frac{1}{\mu_{\text{com}}} \)
Comparison of a Few Cooperative Operation Bounds

- Compare inner bounds
- Bounded by water-filling approach currently
  - Although its not tight

![Comparison of bounds](image)

### Table II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>5 MHz</td>
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<tr>
<td>Center Frequency</td>
<td>3 GHz</td>
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<tr>
<td>Temperature</td>
<td>1000 K</td>
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<tr>
<td>Communications Range</td>
<td>10 km</td>
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<tr>
<td>Communications Power</td>
<td>100 W</td>
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<tr>
<td>Communications Antenna Gain</td>
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<tr>
<td>Communications Receiver Side-lobe Gain</td>
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<tr>
<td>Radar Target Range</td>
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<tr>
<td>Radar Antenna Gain</td>
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<tr>
<td>Radar Power</td>
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<td>Target Process Standard Deviation</td>
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<td>Time-Bandwidth Product</td>
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<tr>
<td>Radar Duty Factor</td>
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</tbody>
</table>
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• Where do we go from here?
Where Are We Going?

• Reuse RF energy for multiple purposes
  – It’s green
  – It’s cognitive

• Lots of room to improve basic theory
  – We are working on a number of issues
  – The bounds work is far from finished
  – Joint information, estimation, and detection theory

• Lots of room for system design and analysis
  – What does real systems look like?
  – Some require simultaneous transmit and receive

• Lots of room to consider partially cooperative approaches
Annulus of “Badness”

Non-Cooperative Implication Simple Example

• Region of range to communications node that maximizes adverse affect

• Assume advanced radar receiver
  – Tries to mitigate interference

• Communications is not radar-aware
  – Vary range to comms transmitter

![Diagram](image.png)
Some of Our Literature


• Upcoming conference papers…

• Submitting journal papers

*Not Really Connected To This Talk, But You Should Buy It Anyway*